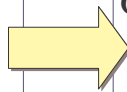




TRAVAUX DIRIGES

Question n°1

- For each of the following transfer functions :
 - find the values of ζ and ω_n ;
 - write, by inspection, the general form of the step response.



$$G(p) = \frac{400}{p^2 + 12p + 400}$$

$$G(p) = \frac{900}{p^2 + 90p + 900}$$

$$G(p) = \frac{225}{p^2 + 30p + 225}$$

$$G(p) = \frac{625}{p^2 + 625}$$

$$G_1(p) = \frac{400}{p^2 + 12p + 400} \quad \begin{cases} K = 1 \\ \omega_n = 20 \text{ rad/s} \\ \zeta = 0,3 \end{cases} \Rightarrow \begin{cases} q(t) = \left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cos(\omega_n t \sqrt{1-\zeta^2} - \psi) \right] u(t) \\ q(t) = [1 - 1,048 e^{-6t} \cos(19,1t - 0,3)] u(t) \end{cases}$$

$$G_2(p) = \frac{900}{p^2 + 90p + 900} \quad \begin{cases} K = 1 \\ \omega_n = 30 \text{ rad/s} \\ \zeta = 1,5 \end{cases} \Rightarrow \begin{cases} q(t) = \left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{\zeta^2-1}} \text{ch}(\omega_n t \sqrt{\zeta^2-1} - \psi) \right] u(t) \\ q(t) = [1 - 0,89 e^{-45t} \text{ch}(33,54t - 1,195)] u(t) \end{cases}$$

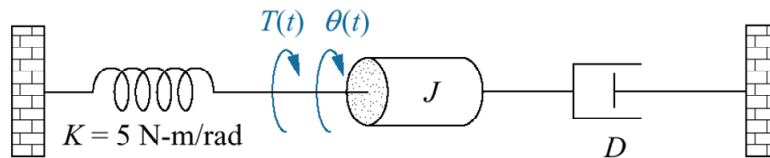
$$G_3(p) = \frac{225}{p^2 + 30p + 225} \quad \begin{cases} K = 1 \\ \omega_n = 15 \text{ rad/s} \\ \zeta = 1 \end{cases} \Rightarrow \begin{cases} q(t) = [1 - e^{-\omega_n t} (1 + \omega_n t)] u(t) \\ q(t) = [1 - e^{-15t} (1 + 15t)] u(t) \end{cases}$$

$$G_4(p) = \frac{625}{p^2 + 625} \quad \begin{cases} K = 1 \\ \omega_n = 25 \text{ rad/s} \\ \zeta = 0 \end{cases} \Rightarrow \begin{cases} q(t) = [1 - \cos \omega_n t] u(t) \\ q(t) = [1 - \cos 25t] u(t) \end{cases}$$

Question n°2

Question 2 - 1: For $G(p) = \frac{100}{p^2 + 15p + 100}$
find $D1\%$, t_p , $tr5\%$ et $tr2\%$

Question 2 - 2: Given the system shown in figure find J and D
to yield 20% overshoot and a settling time ($tr2\%$)
of 2 seconds for a step input of torque $T(t)$.



Question 2 - 1:

$$\omega_n = 10 \text{ rad/s} \quad \zeta = 0,75$$

$$D1\% = 100 \cdot \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 2,84\% \quad t_p = \frac{\pi}{\sqrt{1-\zeta^2}} = 0,475 \text{ s}$$

$$tr5\% \approx \frac{3}{\omega_n} = 0,3 \text{ s} \quad tr2\% \approx \frac{4}{\zeta\omega_n} = 0,53 \text{ s}$$

Question 2 - 2:

$$D1\% = 20\% \quad tr2\% = 2 \text{ s} \quad K = 5 \text{ Nm/rad}$$

$$G(p) = \frac{\theta}{T}(p) = \frac{1}{Jp^2 + Dp + K} \Rightarrow \begin{cases} \omega_n = \sqrt{K/J} \\ \zeta = D/2\sqrt{KJ} \Rightarrow \zeta^2 = D^2/4KJ \end{cases}$$

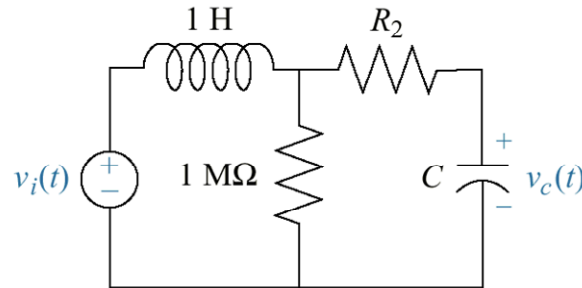
$$D1 = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0,2 \Rightarrow \left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \ln(0,2) \Rightarrow \zeta^2 = 0,208$$

$$\left. \begin{aligned} \zeta^2 = \frac{D^2}{20J} = 0,208 \\ tr2\% \approx \frac{4}{\zeta\omega_n} = \frac{8J}{D} = 2 \end{aligned} \right\} \Rightarrow \begin{cases} D = 1,04 \text{ N.m.s/rad} \\ J = 0,26 \text{ kg.m}^2 \end{cases}$$

Question n°3a : A traiter par les ING2 TIE



For the circuit shown in figure below, find the value of R_2 and C to yield 15% overshoot with a settling time ($\text{tr}2\%$) of 1 ms for a voltage across the capacitor with $v_i(t)$ as a step input.



$$G(p) = \frac{V_o(p)}{V_i(p)} = \frac{1}{Cp} \frac{\frac{R1}{R1 + Lp}}{\frac{R1Lp}{R1 + Lp} + R2 + \frac{1}{Cp}} = \frac{R1}{CR1Lp^2 + R2Cp(R1 + Lp) + (R1 + Lp)}$$

$$G(p) = \frac{1}{(LC + \tau_L \tau_C)p^2 + (\tau_L + \tau_C)p + 1} \quad \text{avec} \quad \begin{cases} \tau_C = CR2 \\ \tau_L = \frac{L}{R1} \end{cases}$$

$$G(p) = \frac{1}{\frac{p^2}{\omega_n^2} + \frac{2\zeta}{\omega_n}p + 1} \quad \begin{cases} \omega_n^2 = \frac{1}{LC + \tau_L \tau_C} \\ \frac{2\zeta}{\omega_n} = (\tau_L + \tau_C) \end{cases}$$

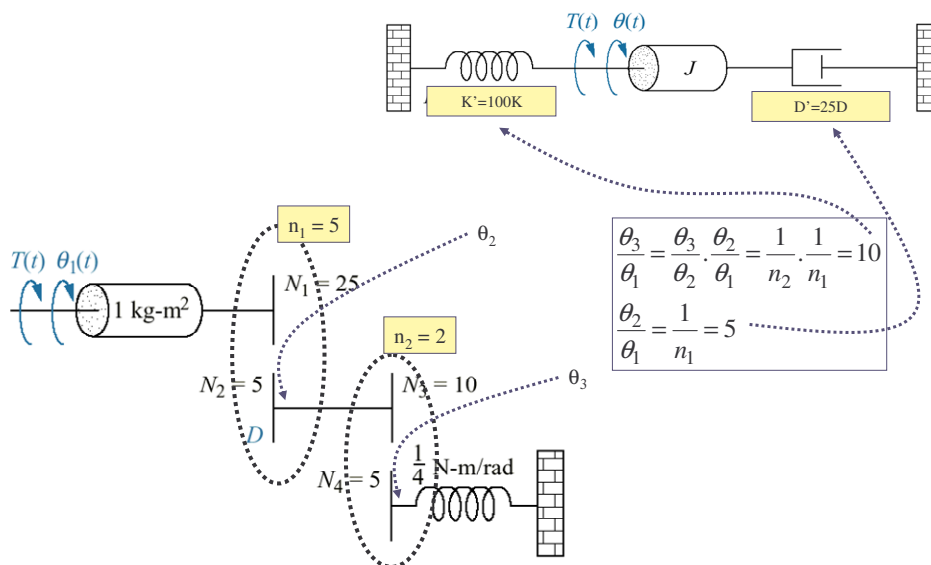
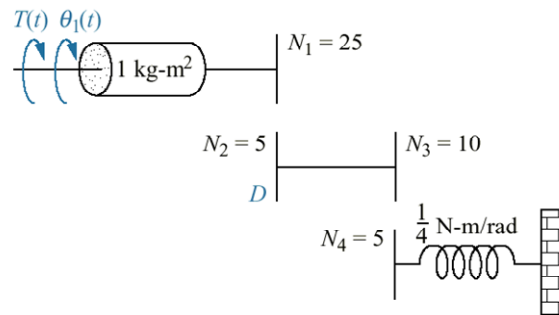
$$D1 = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0,15 \Rightarrow \left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \ln(0,15) \Rightarrow \zeta^2 = 0,267$$

$$\left. \begin{aligned} \zeta^2 &= \frac{(\tau_L + \tau_C)^2}{4(LC + \tau_L \tau_C)} = 0,267 \\ \text{tr}2\% \approx \frac{4}{\zeta\omega_n} &= \frac{8(LC + \tau_L \tau_C)}{(\tau_L + \tau_C)} = 10^{-3} \end{aligned} \right\} \Rightarrow \begin{cases} C = 16,7 \text{ nF} \\ R2 = 7,93 \text{ k}\Omega \end{cases}$$

Question n°3b : A traiter par les ING2 ME



Given the system shown in figure below, find the damping D , to yield a 30% overshoot in output angular displacement for a step input in torque.



$$D1\% = 30\% \quad J = 1 \text{ kg.m}^2 \quad K' = 100K = 25 \text{ Nm/rad}$$

$$G(p) = \frac{\theta_1}{T}(p) = \frac{1}{Jp^2 + D'p + K'} \Rightarrow \begin{cases} \omega_n = \sqrt{K'/J} \\ \zeta = D'/2\sqrt{K'J} \Rightarrow \zeta^2 = D'^2/4K'J \end{cases}$$

$$D1 = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0,3 \Rightarrow \left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \ln(0,3) \Rightarrow \zeta^2 = 0,128$$

$$\zeta^2 = \frac{D'^2}{100} = 0,128 \Rightarrow D' = 3,58 \text{ N.m.s/rad} \Rightarrow D = 0,143 \text{ N.m.s/rad}$$

Question n°4 : Lieux de transfert

- Donner l'allure des lieux de Nyquist et de Black-Nichols et tracer les diagrammes de Bode asymptotiques des fonctions de transfert suivantes :

$$H_1(p) = \frac{1}{p}$$

$$H_2(p) = \frac{5}{p}$$

$$H_3(p) = \frac{e^{-0,5p}}{p}$$

$$H_4(p) = \frac{1}{p^2}$$

$$H_5(p) = \frac{16}{p^2}$$

$$H_6(p) = \frac{1}{p(1+p)}$$

$$H_7(p) = \frac{9}{p(1+5p)}$$

$$H_8(p) = \frac{9(1+p)}{p^2(1+5p)}$$

• Script MALAB

```
% Script Au41_TD2_0506.m
% Cours Au 41 de J.-L. Cougnon
% Version du 11 novembre 2005
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;clc;
H1=tf(1,[1 0])
H2=5*H1
H3=zpk([], [0], 1, 'iodelaymatrix', 0.5)
H4=H1*H3
H5=16*H4
H6=tf(1,[1 1 0])
H7=tf(9,[5 1 0])
H8=9*tf([1 1],[5 1 0 0])
w=logspace(-1,1);

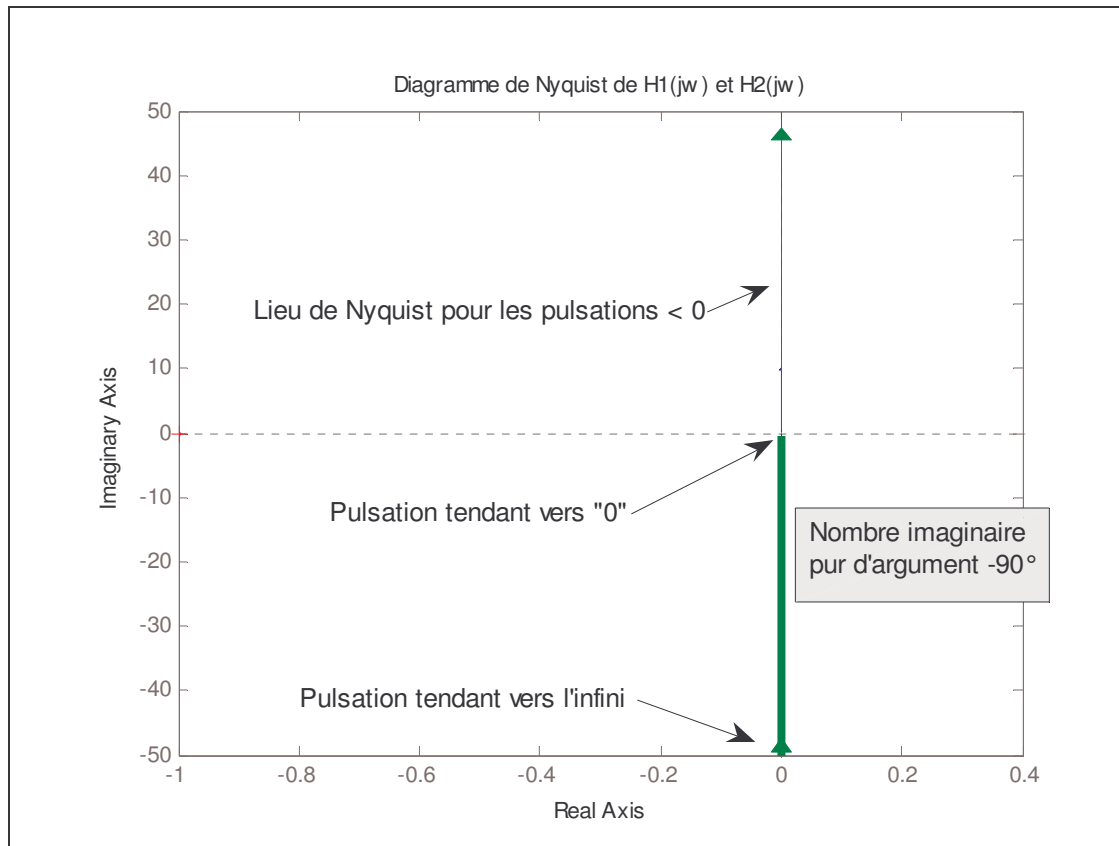
figure(1);nyquist(H1,H2,w)
title('Diagramme de Nyquist de H1(jw) et H2(jw)')
figure(2);bode(H1,H2,w);grid
title('Diagramme de Bode de H1(jw) et H2(jw)')
figure(3);nichols(H1,H2,w);grid
title('Diagramme de Black-Nichols de H1(jw) et H2(jw)')

figure(4);nyquist(H1,H3,w)
title('Diagramme de Nyquist de H1(jw) et H3(jw)')
figure(5);bode(H1,H3,w);grid
title('Diagramme de Bode de H1(jw) et H3(jw)')
figure(6);nichols(H1,H3,w);grid
title('Diagramme de Black-Nichols de H1(jw) et H3(jw)')

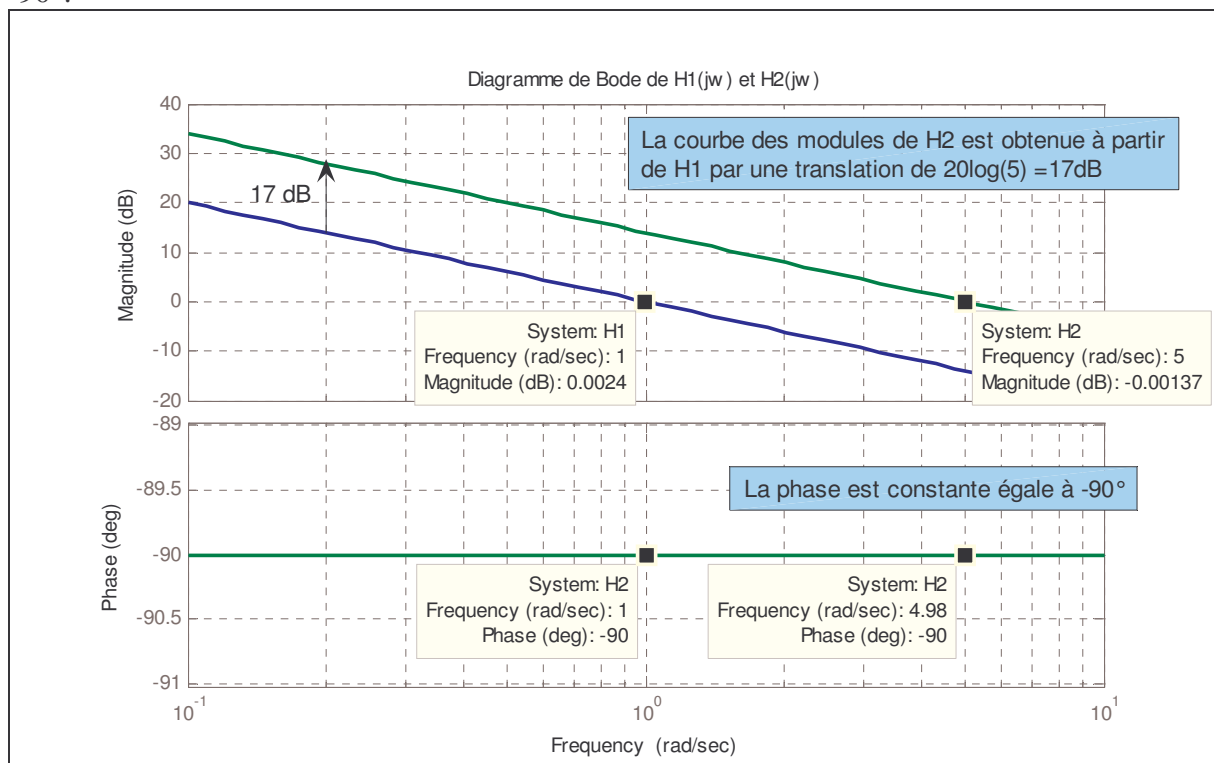
figure(7);bode(H4,H5,w);grid
title('Diagramme de Bode de H4(jw) et H5(jw)')

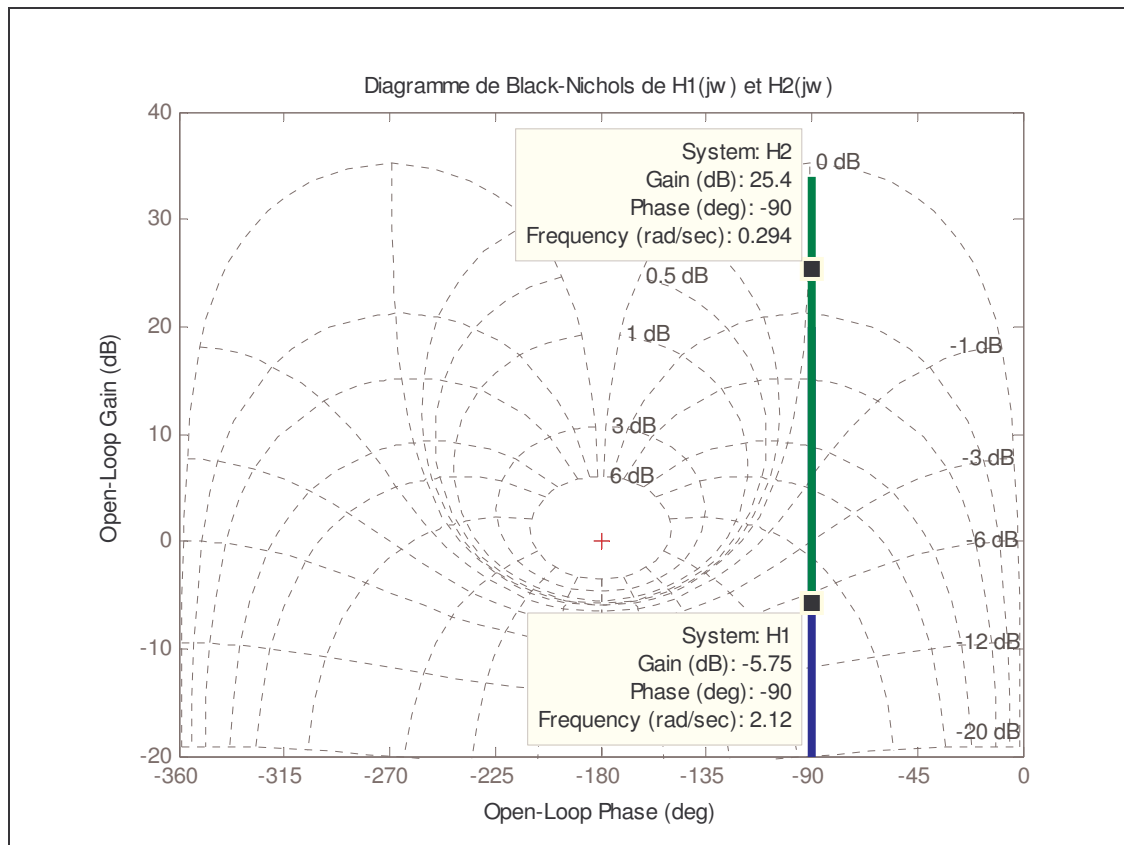
figure(8);bode(H6,H7,H8,w);grid
title('Diagramme de Bode de H6(jw), H7(jw) et H8(jw)')
figure(9);nichols(H6,H7,H8,w);grid
title('Diagramme de Black-Nichols de H6(jw), H7(jw) et H8(jw)')
```

- Traçons $H1$ et $H2$

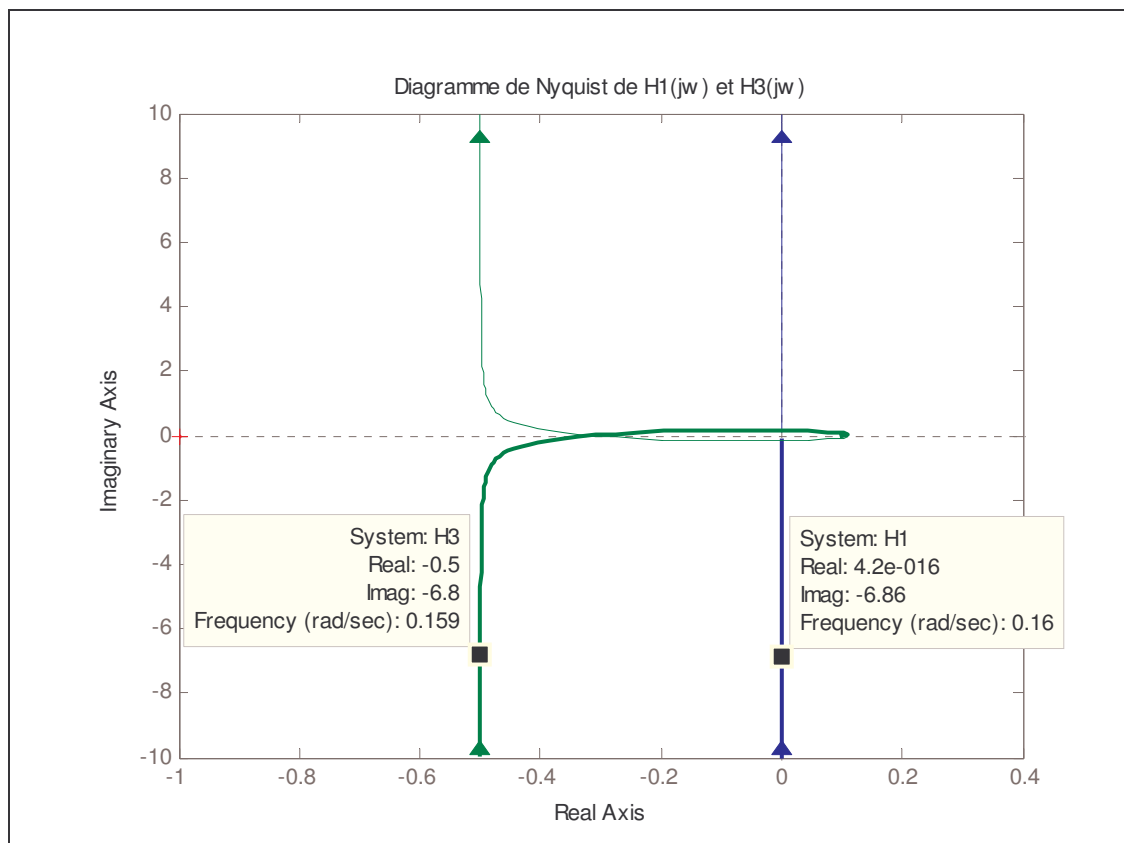


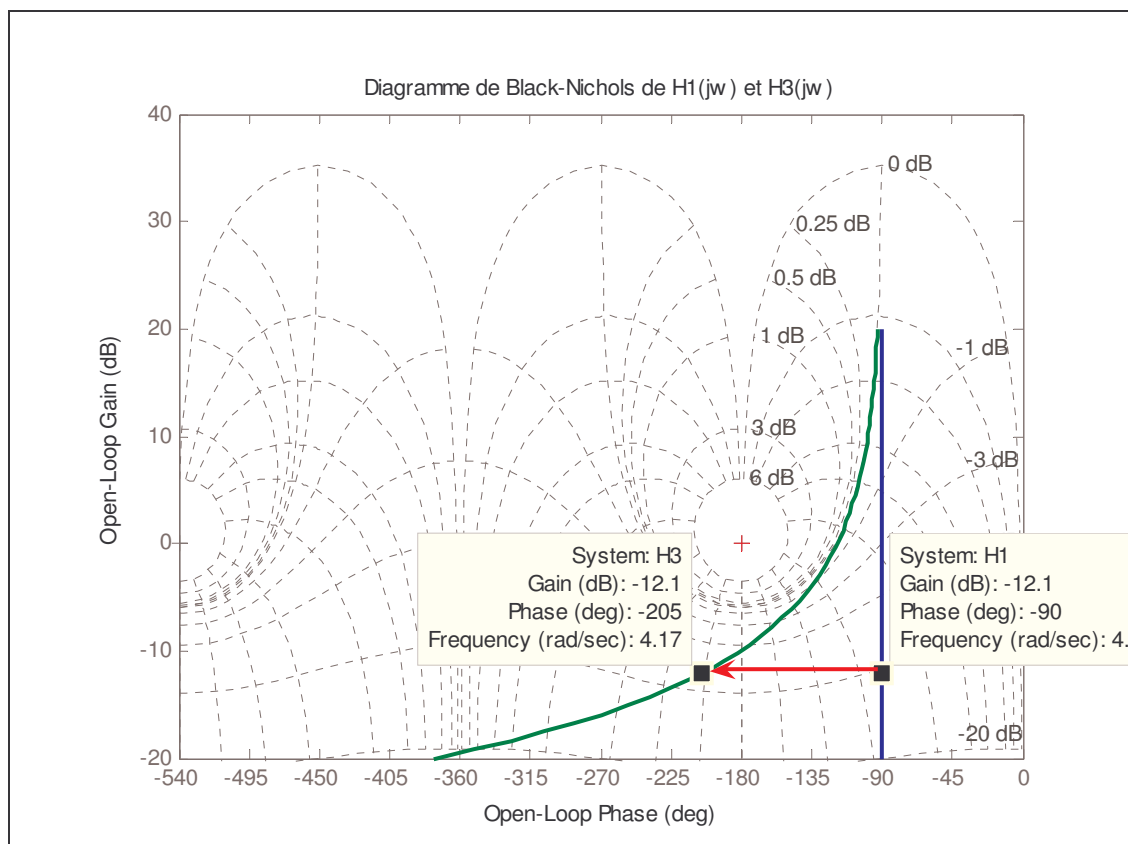
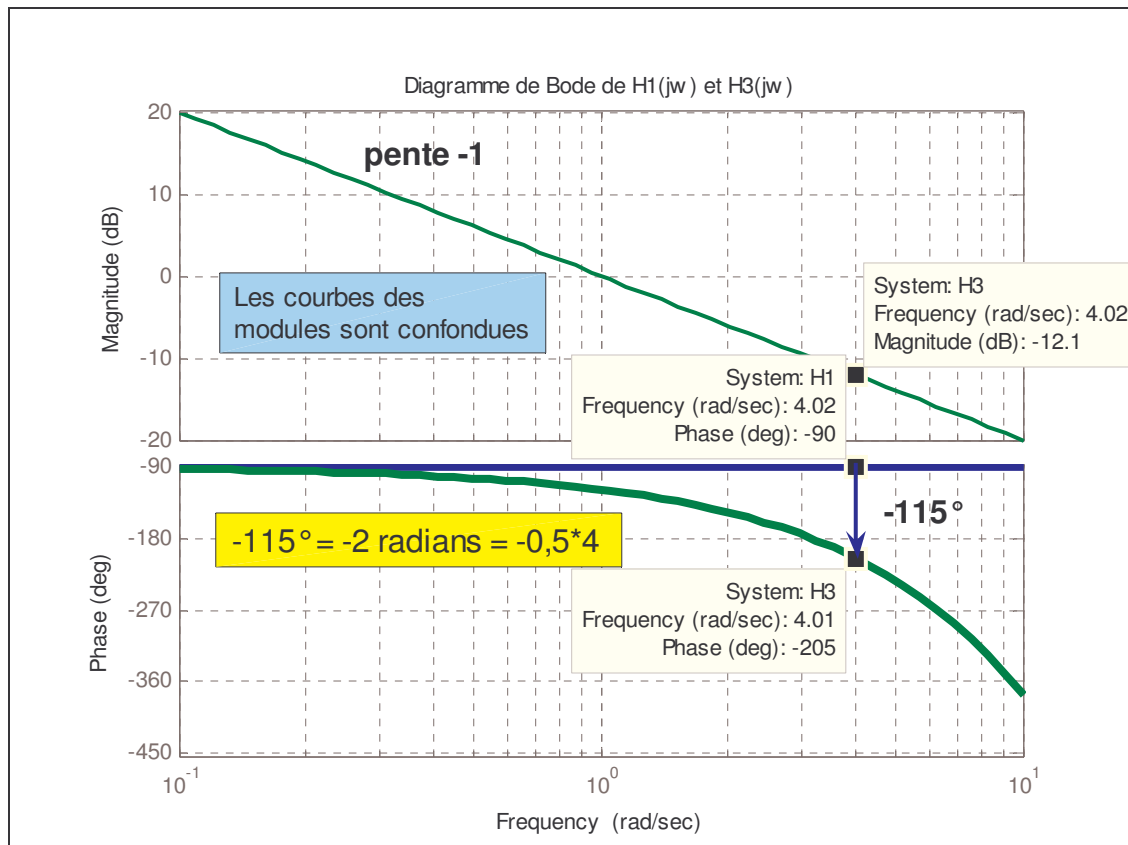
$H1(j\omega)$ et $H2(j\omega)$ sont des nombres complexes purs ; c'est ce que l'on observe sur le lieu de Nyquist. Attention ce tracé est donné pour une pulsation variant de $-\infty$ à $+\infty$. Lorsque la pulsation varie de 0 à l'infini le module varie de l'infini à 0 ; l'argument reste constant égal à -90° .





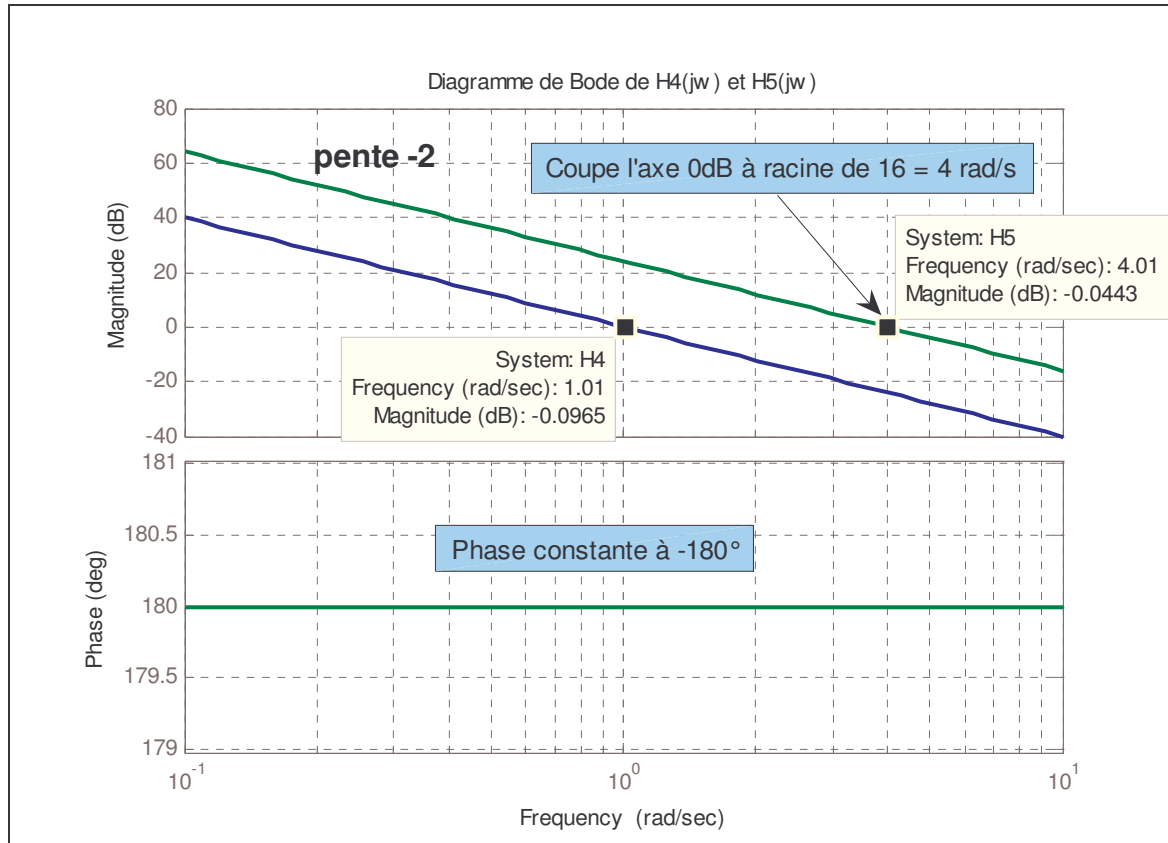
- **Traitons H3 :**



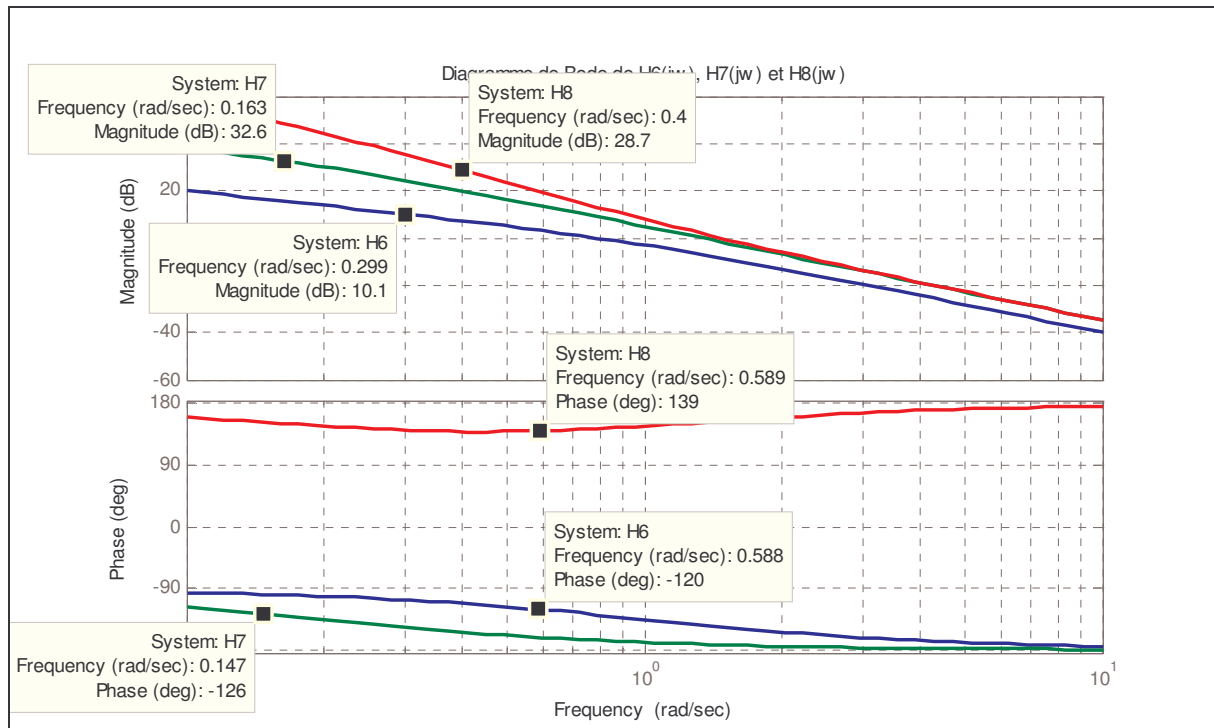


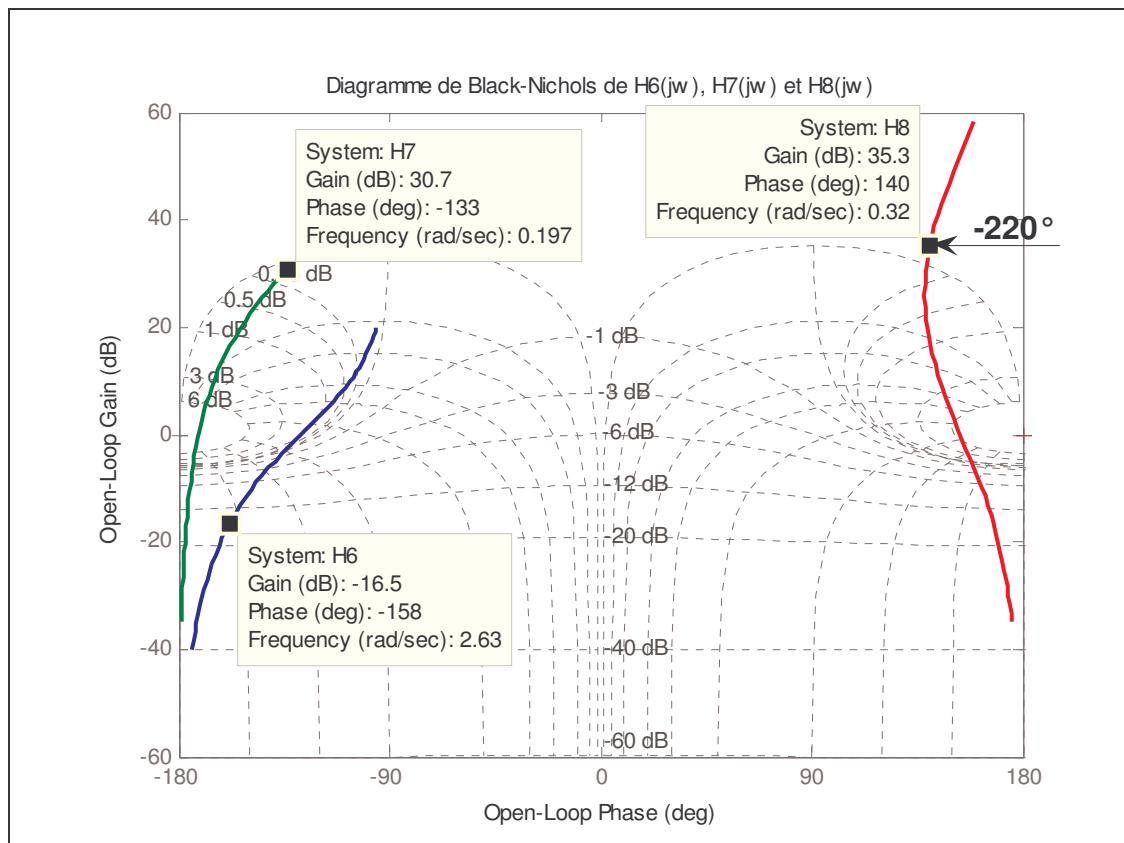
Toutes ces figures mettent en évidence l'effet du retard sur la phase.

- Traçons H4 et H5 (double intégrateur \Rightarrow pente -2).



- Traçons H6, H7 et H8.





Nos élèves veilleront à réaliser un tracé asymptotique des diagrammes de Bode.
Il conviendra d'exploiter avec intelligence les courbes de phase. Ainsi à la pulsation de $0,32\text{rad/s}$ la phase de $H8(j\omega)$ est égale -220° .

Question n°5 :

Calculer K , ζ , ω_n , ω_R , Q pour les transmittances suivantes :

$$H_9(p) = \frac{10}{\frac{p^2}{100} + 0,05p + 1}$$

$$H_{10}(p) = \frac{40}{p^2 + 12p + 20}$$

$$H_9(p) = \frac{10}{\frac{p^2}{100} + 0,05p + 1} \quad \left\{ \begin{array}{l} K = 10 \\ \omega_n = 10 \text{ rad/s} \\ \zeta = 0,25 \Rightarrow \zeta < 0,7 \Rightarrow \text{résonance} \\ \omega_R = \omega_n \sqrt{1 - 2\zeta^2} = 9,35 \text{ rad/s} \\ Q = \frac{1}{2\zeta \sqrt{1 - \zeta^2}} = 1,93 \text{ (soit 5,74dB)} \end{array} \right.$$

$$H_{10}(p) = \frac{40}{p^2 + 12p + 20} \quad \left\{ \begin{array}{l} K = 2 \\ \omega_n = \sqrt{20} \text{ rad/s} = 2\sqrt{5} \text{ rad/s} \\ \zeta = \frac{3}{\sqrt{5}} > \frac{\sqrt{2}}{2} \Rightarrow \text{pas de résonance} \end{array} \right.$$

Question n°6 :

- Donner l'allure du lieu de Nyquist et tracer le lieu de Bode asymptotique des transmittances ci contre.



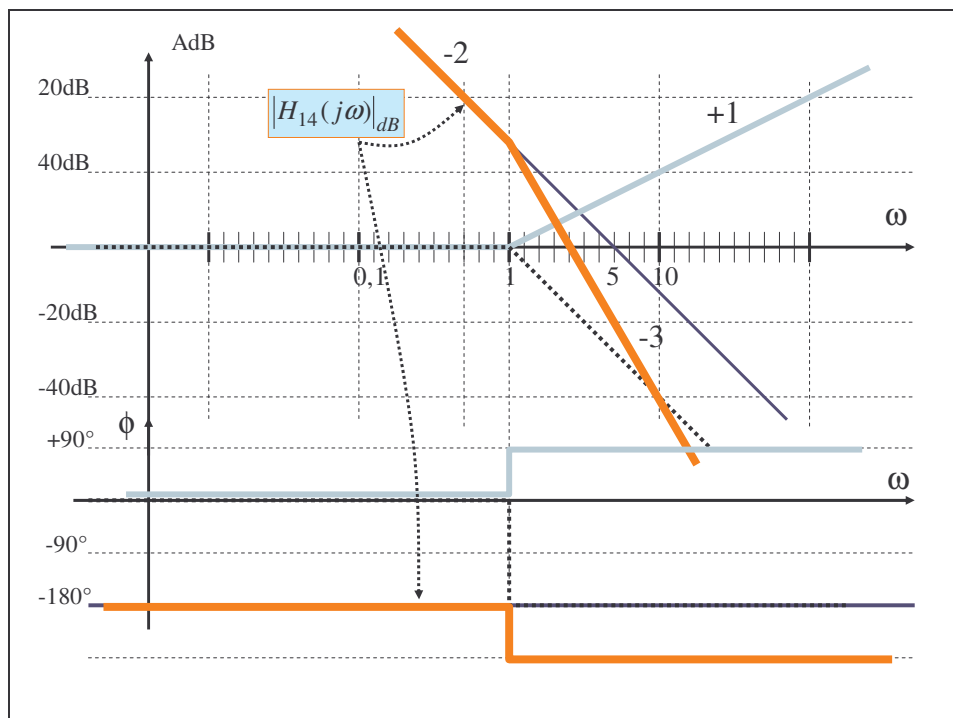
$$H_{11}(p) = \frac{10}{10p^2 + 5p + 10}$$

$$H_{12}(p) = \frac{(1+p)}{(p^2 + 0,5p + 1)}$$

$$H_{13}(p) = \frac{(1+p)}{p(p^2 + 0,5p + 1)}$$

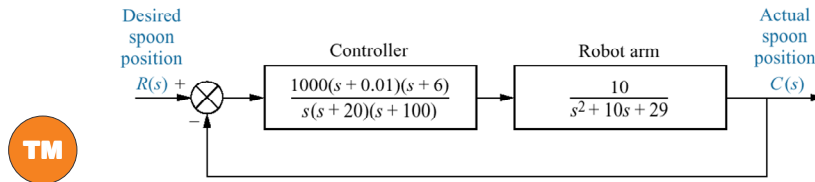
$$H_{14}(p) = \frac{25(1+p)}{p^2(p^2 + 0,5p + 1)}$$

Nous avons traité ci dessous le tracé asymptotique de $H_{14}(p)$.



Question n°8 :

A robot arm can be used to feed people with disabilities. The control system guides the spoon to the food and then to a position near the person's mouth. The arm uses a special pneumatically controlled actuator. Assume the simplified block diagram shown in figure below for regulating the spoon at a distance from the mouth.



Write a program in MATLAB that will do the following :

- Allow a value of gain K to be entered from the keyboard;
- Display the Bode plots of open-loop and closed-loop;
- Plot the closed-loop step response;
- Determine the range of K for stability.

```
%
% Script Au41_TD2_0506_Q8.m
% Cours Au 41 de J.-L. Cougnon
% Version du 28 novembre 2005
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;clc;
disp('Fonction de transfert du
correcteur =')
Cdp=zpk([-0.01 -6],[0 -20 -100],1000)
disp('Fonction de transfert du robot =')
Rdp=tf([10],[1 10 29])
K=input('Gain K = ')
ftbo=K*Cdp*Rdp
ftbf=feedback(ftbo,1)
figure(1);step(ftbf,30)
% Par itération on trouve Kosc=30.
% On vérifie que la FTBF a 2 pôles
% à partie réelle nulle.
% La pulsation des auto oscillations
% est de 50 rad/s.
[z,p,k]=zpkdata(ftbf,'v')
figure(2);bode(ftbo,ftbf);grid
title('Diagrammes de Bode de la FTBO et
de la FTBF pour K=1')
figure(3);step(ftbf,10)
title('Réponse indicielle de la FTBF
pour K=1')
```

Fonction de transfert du correcteur =

Zero/pole/gain:
 $1000 (s+0.01) (s+6)$

 $s (s+20) (s+100)$

Fonction de transfert du robot =

Transfer function:
 10

 $s^2 + 10 s + 29$

Gain $K = 30$

$K = 30 \Rightarrow$ **Valeur de K_{osc}**

Zero/pole/gain:
 $300000 (s+0.01) (s+6)$

$s (s+20) (s+100) (s^2 + 10s + 29)$

Zero/pole/gain:
 $300000 (s+6) (s+0.01)$

$(s+124) (s+6.022) (s+0.009689) (s^2 - 0.05141s + 2488)$

$z =$
 -6.0000
 -0.0100

$p =$
 $1.0e+002 *$
 -1.2402

$0.0003 + 0.4988i \Rightarrow$ **pôle à partie réelle nulle**
 $0.0003 - 0.4988i$
 -0.0001
 -0.0602

$k =$
 300000

Pour $K = 1$ on obtient :

