Formes canoniques

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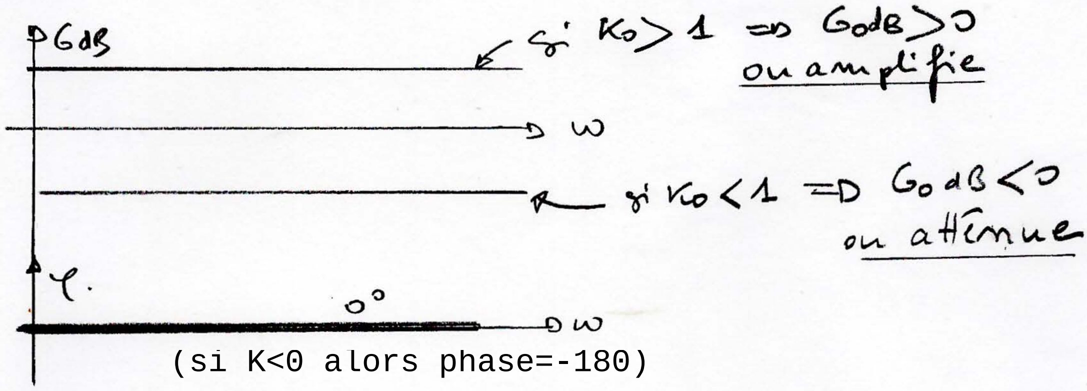
I - 1 - $\text{Arg } \underline{H_0} = \text{Arg } K_0 = 0°$ → indépendant de ω !
(avec Ko>0)

2 - $|H_0| = K_0$ et $G_{0dB} = 20 \log |H_0(j\omega)|$
$= 20 \log K_0$ → indépendant de ω !

3. Allure Bode.



(si K<0 alors phase=-180)

4 - $\text{Arg } \underline{H_1}(j\omega) = \text{Arg}\left(j\frac{\omega}{\omega_0}\right) = \frac{\pi}{2} \text{ rad.}$
$= 90°$

5 - $|H_1(j\omega)| = \left|j\frac{\omega}{\omega_0}\right| = \frac{\omega}{\omega_0} \Rightarrow G_{1dB} = 20 \log\left(\frac{\omega}{\omega_0}\right)$

6 - rappel $\log \frac{a}{b} = \log a - \log b$

$\Rightarrow G_{1dB} = \underbrace{20}_{a} \log(\omega) \underbrace{- 20 \log \omega_0}_{b}$

7 - représentation sous forme d'un Bode (semi-log)

axe x : $\omega \longmapsto \underset{[cm]}{l_\omega} = \underset{[cm/rd]}{k_x} \times \log(\omega) + 0$ si origine des $\omega = 1$

axe y $G_{dB} \longmapsto \underset{[cm]}{l_{G_1}} = \underset{[cm/dB]}{k_y} \times G_{1dB} + 0$ si origine des dB à $\emptyset$.

Comme $G_{1dB} = a \log\omega + b \Rightarrow \underline{l_{G_1} = a\frac{k_y}{k_x} l_\omega + k_y b}$

$\frac{l_{G_1}}{k_y}$ $\qquad \frac{l_\omega}{k_x}$

équation d'une droite
point à l'origine à $b = -20 \log \omega_0$
$a = +20$
pente

FI

Rq : prenons 1 exemple pour comprendre a

$G(\omega = \omega_0) = a \log \omega_0 - a \log \omega_0 = 0$ dB.

1 décade + loin

$G(\omega = 10\omega_0) = a \log(10\omega_0) - a \log \omega_0$

$= a \log 10 + a \log \omega_0 - a \log \omega_0$

$= a \log 10 = a = +20$.

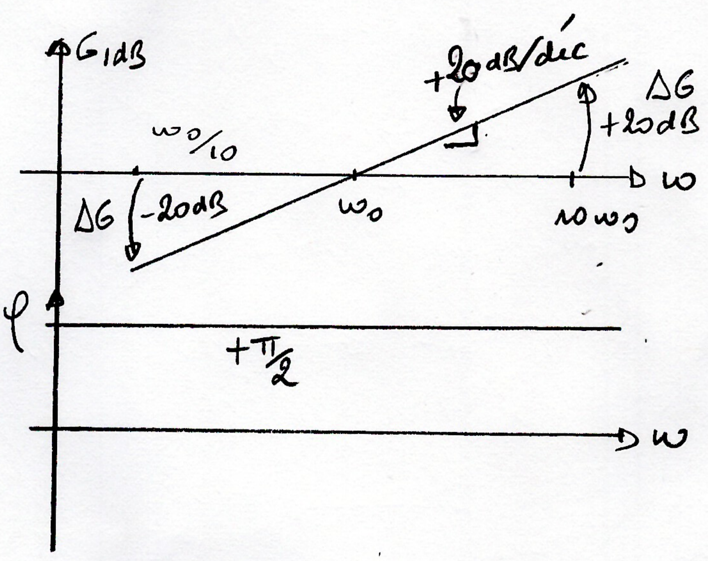
$\Delta G = G(10\omega_0) - G(\omega_0) = +20$ dB

On dit que la pente est de +20dB/décade.

↳ par rapport à ω_0

- si $\omega > \omega_0$ le signal est amplifié : $G > 0$ et $A > 1$.
- si $\omega < \omega_0$ le signal est atténué : $G < 0$ et $A < 1$

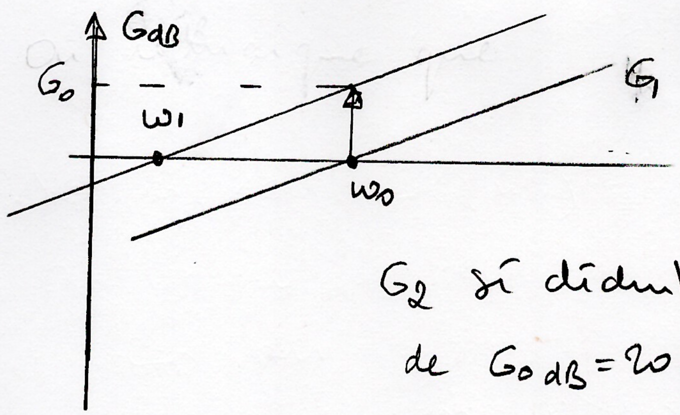
Allure du Bode



8 - $\underline{H}_2(j\omega) = K_0 j\frac{\omega}{\omega_0} = j\frac{\omega}{\frac{\omega_0}{K_0}} = j\frac{\omega}{\omega_1}$ avec $\omega_1 = \frac{\omega_0}{K_0}$

$G_2(\omega) = 20\log\frac{\omega}{\frac{\omega_0}{K_0}}$ Pour $\omega = \omega_0$ $G_2(\omega_0) = K_0$

$\omega = \omega_1$ $G_2(\omega_1) = 0$ dB.



G_2 se déduit de G_1 par translation

de $G_{0dB} = 20\log K_0$ (ici >0 pour l'exemple)

Démonstration : $|H_2| = |K_0| \times \left|\frac{\omega}{\omega_0}\right|$

$= |H_0| \times |H_1|$

$\Rightarrow$ $G_2 = 20\log|H_2| = 20\log|H_0| \times |H_1|$

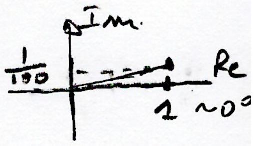
$= 20\log|H_0| + 20\log|H_1|$

$= G_{0dB} + G_{1dB}$

formes canoniques : 1° ordre

1 - $\text{Arg } H_3 = \varphi = \text{Arg}\left(1 + j\frac{\omega}{\omega_3}\right)$

$= \text{Atan}\left(\frac{\omega}{\omega_3}\right)$ Rq : Atan ou $\tan^{-1}$ ou arctan

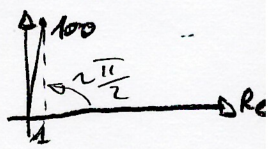


2 - si $\omega \ll \omega_3 \Rightarrow \text{Arg } H_3 \to \text{Arg}(1 + j0)$

$\Rightarrow \varphi \to 0.$

si $\omega \gg \omega_3 \Rightarrow \text{Arg } H_3 \simeq j\frac{\omega}{\omega_3}$ car $\left|\frac{\omega}{\omega_3}\right| \gg 1$

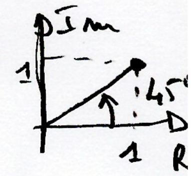
$\varphi \to \frac{\pi}{2}$



Rq : Point particulier (non demandé)

si $\omega = \omega_0$ alors $\text{Arg } H_3 = \text{Arg}(1 + j)$

$\varphi = 45° = \frac{\pi}{4}$ rd.



3 - $|H_3(j\omega)| = \sqrt{1^2 + \left(\frac{\omega}{\omega_3}\right)^2}$

$G_{3dB} = 20 \log |H_3| = 20 \log \left[1^2 + \left(\frac{\omega}{\omega_3}\right)^2\right]^{\frac{1}{2}}$

$= 10 \log \left[1^2 + \left(\frac{\omega}{\omega_3}\right)^2\right]$

4 - si $\omega \ll \omega_3$ alors $\left(\frac{\omega}{\omega_3}\right)^2 \ll 1^2$

$\Rightarrow |H_3| \sim 1 \Rightarrow G_{3dB} \to 0\,dB$

si $\omega \gg \omega_3$ alors $\left(\frac{\omega}{\omega_3}\right)^2 \gg 1^2$

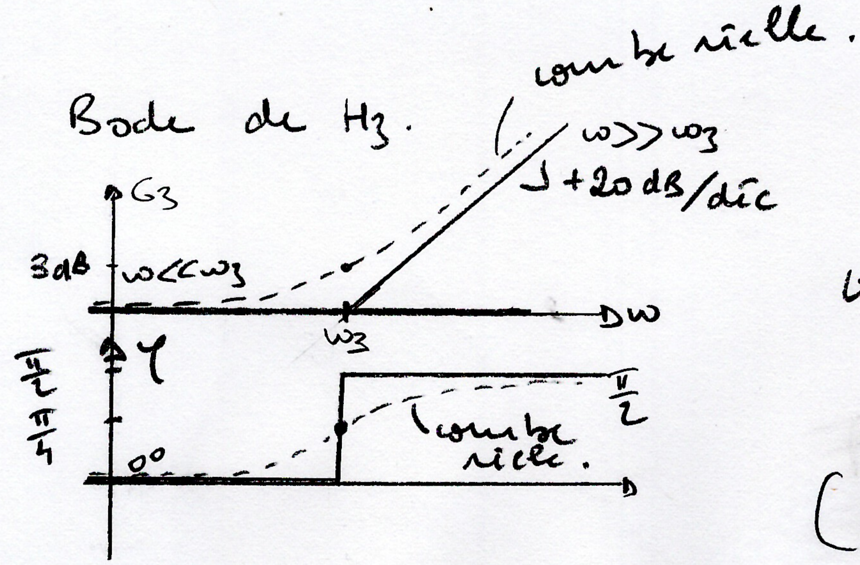
$|H_3| \sim \frac{\omega}{\omega_3} \Rightarrow G_{3dB} \sim 20 \log \frac{\omega}{\omega_3}$

se comporte comme 1 dérivateur à haute fréquence

Rq : Point particulier : $\omega = \omega_3$

$\Rightarrow |H_3| = \sqrt{2}$ et $G_{3dB} = 20 \log \sqrt{2} = +3\,dB$

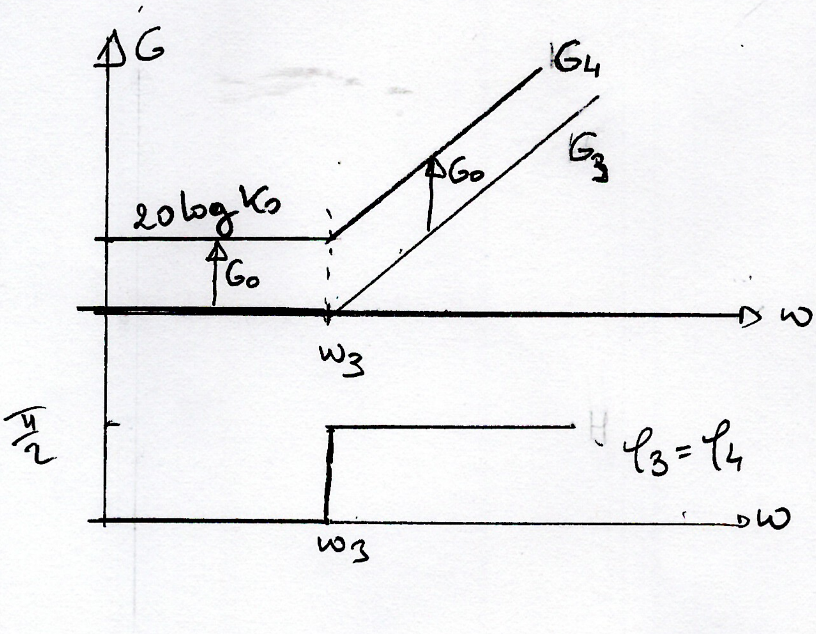
5 - Bode de H_3.



comportement global
filtre passe haut
(pseudo dérivateur)

6 - $G_{4dB} = G_{0dB} + G_{3dB}$

$$\mathrm{Arg}\, \underline{H_4} = \underbrace{\mathrm{Arg}\, \underline{H_0}}_{=0°} + \mathrm{Arg}\, \underline{H_3} = \mathrm{Arg}\, \underline{H_3}$$



III Combinaisons.

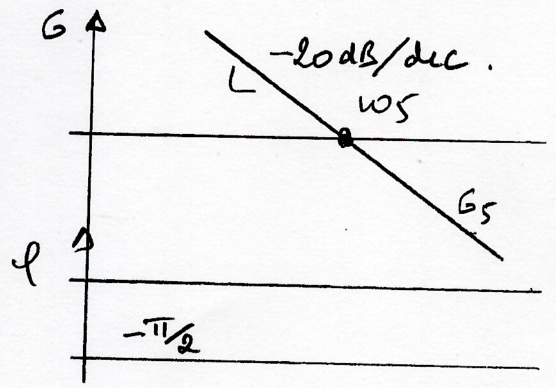
- $\underline{H_5} = \dfrac{K_5}{j\omega}$ = On cherche à mettre sous la forme

$$\frac{1}{j\frac{\omega}{\omega_0}} = \frac{\omega_0}{j\omega} = \frac{1}{\underline{H_1}}$$

$$\underline{H_5} = K_5 \times \frac{\omega_0}{j\omega} = \frac{K_5\omega_0}{j\omega} = \frac{\omega_5}{j\omega} = \frac{1}{j\frac{\omega}{\omega_5}} \quad \text{avec } \omega_5 = 1 \times K_5$$

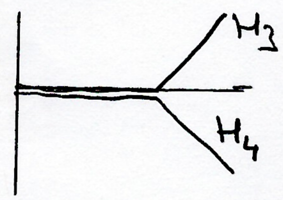
rad.

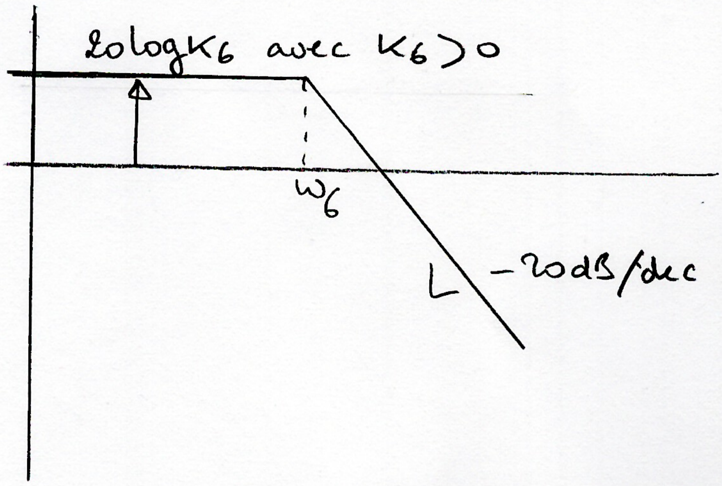
$\underline{d}$: forme "inverse" de $\underline{H_2}$ (ou $\underline{H_1}$)



C'est un intégrateur pur.

- $\underline{H_6} = \dfrac{K_6}{1 + j\frac{\omega}{\omega_6}} = \underbrace{K_6}_{\text{gain pur = décalage.}} \times \dfrac{1}{1 + j\frac{\omega}{\omega_6}}$ — Forme inverse de H_3!

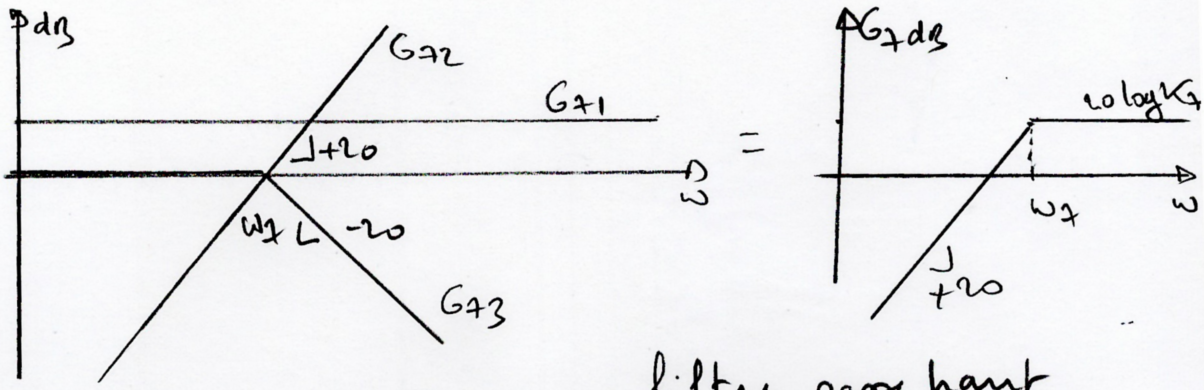




C'est un pseudo intégrateur (filtre passe BAS)

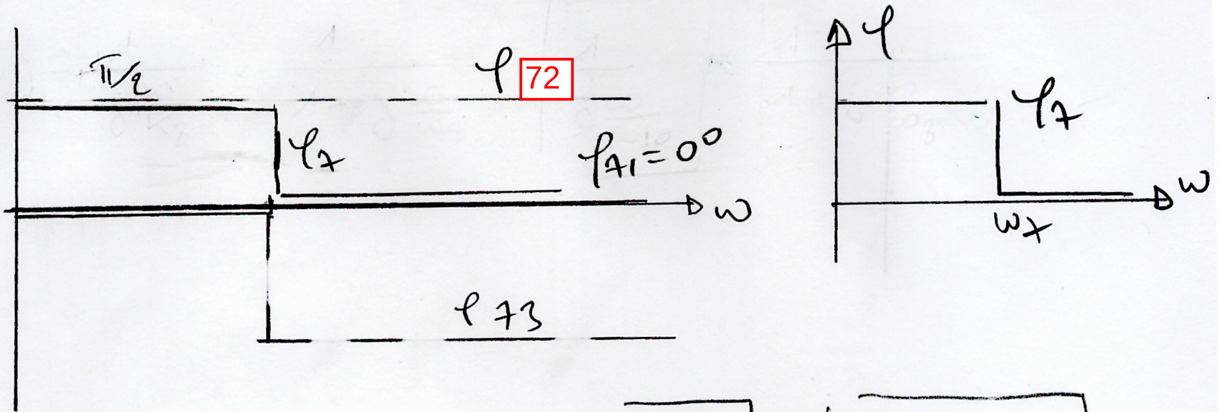
• $H_7(j\omega) = K_7 \times \dfrac{j\frac{\omega}{\omega_7}}{1+j\frac{\omega}{\omega_7}} = \boxed{K_7} \times \boxed{j\frac{\omega}{\omega_7}} \times \boxed{\dfrac{1}{1+j\frac{\omega}{\omega_7}}}$

$G_{7\,dB} = G_{71} + G_{72} + G_{73}.$



filtre passe haut
(pseudo dérivateur)

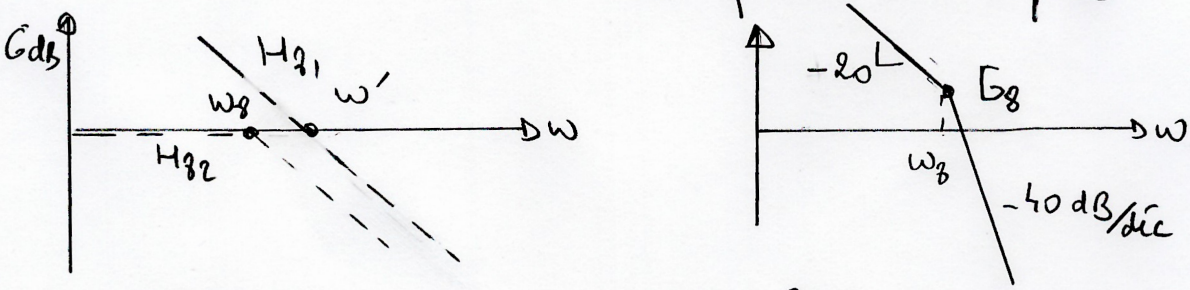
Arg H_7 =
Arg H_{71} +
Arg H_{72} +
Arg H_{73}

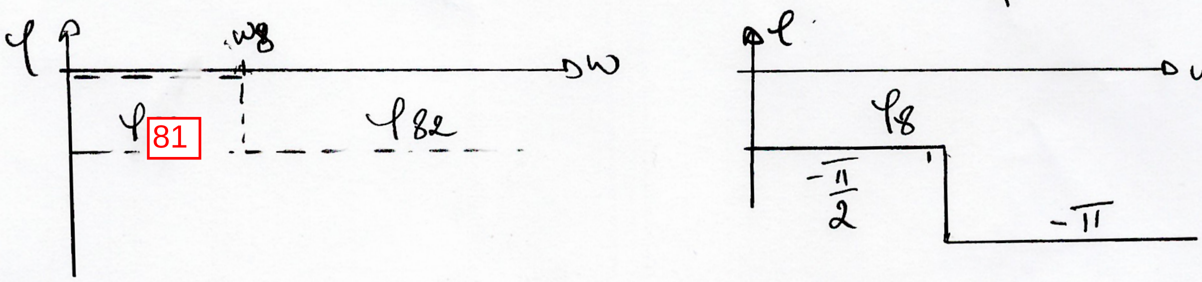


• $H_8(j\omega) = \dfrac{1}{j\frac{\omega}{K_8\times 1}} \times \dfrac{1}{1+j\frac{\omega}{\omega_8}} = \boxed{\dfrac{1}{j\frac{\omega}{\omega'}}} \times \boxed{\dfrac{1}{1+j\frac{\omega}{\omega_8}}}$

H_{81} H_{82}

formes canoniques

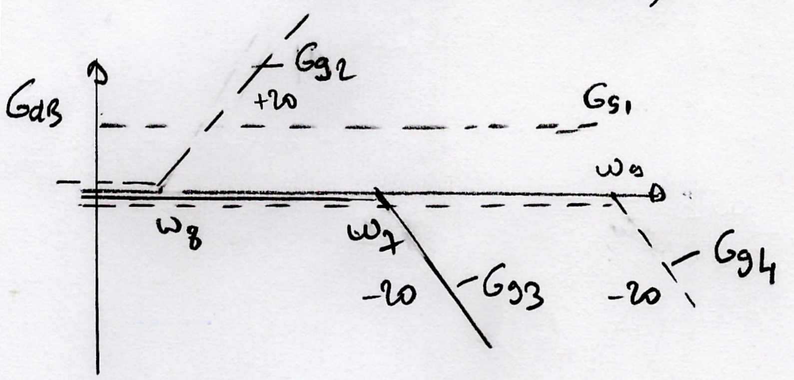


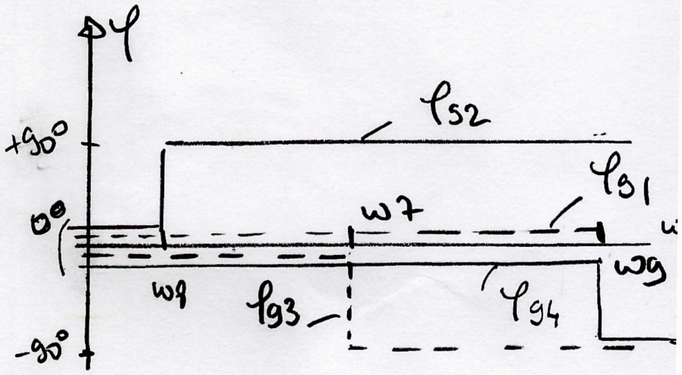


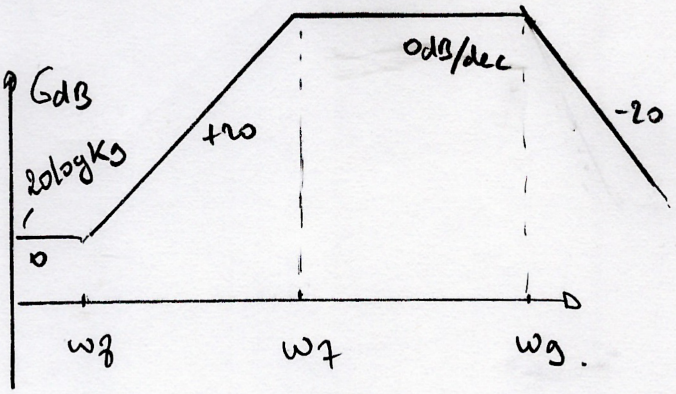
5 - $$H_g(j\omega) = \underbrace{K_g}_{H_{g1}} \times \underbrace{\left(1+j\frac{\omega}{\omega_8}\right)}_{H_{g2}} \times \underbrace{\frac{1}{\left(1+j\frac{\omega}{\omega_7}\right)}}_{H_{g3}} \times \underbrace{\frac{1}{\left(1+j\frac{\omega}{\omega_9}\right)}}_{H_{g4}}$$

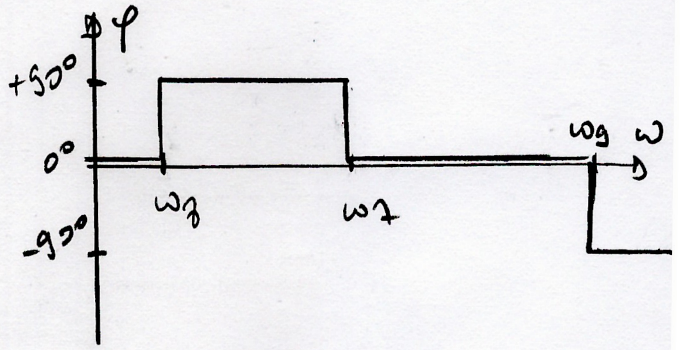
avec $\omega_8 << \omega_7 << \omega_9$.

(Rq : l'ordre des pulsations de coupure influence le résultat !!)







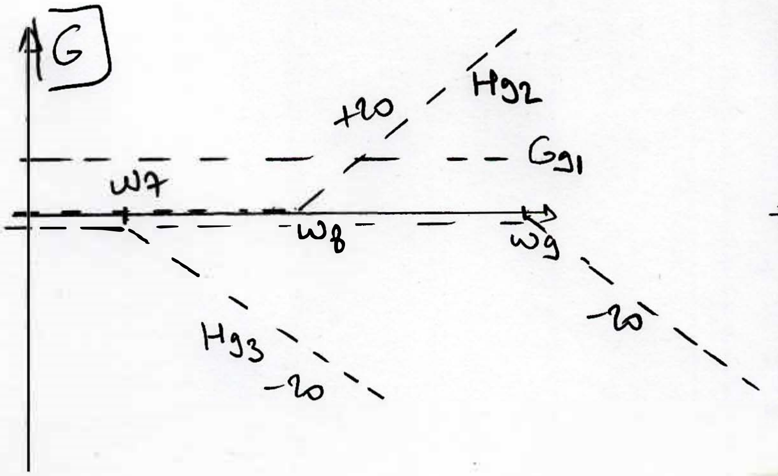


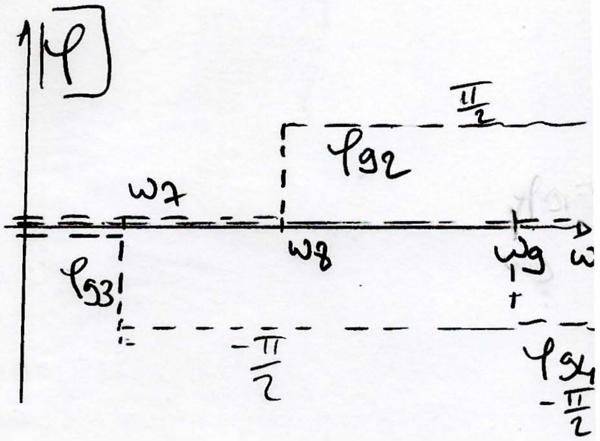
Bande passante

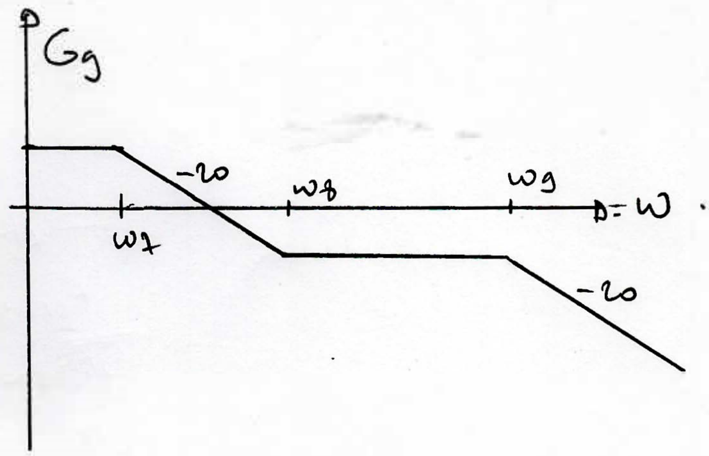
c'est 1 filtre passe bande

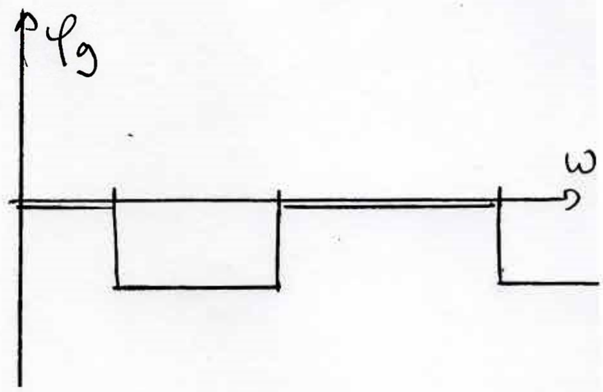
Bonus: etude lorsque w7<<w8<<w9

5 $H_g(j\omega) = \underbrace{K_g}_{H_{g1}} \times \underbrace{\left(1+j\frac{\omega}{\omega_8}\right)}_{H_{g2}} \times \underbrace{\frac{1}{\left(1+j\frac{\omega}{\omega_7}\right)}}_{H_{g3}} \times \underbrace{\frac{1}{\left(1+j\frac{\omega}{\omega_9}\right)}}_{H_{g4}}$









Rg : • Si $\omega_7 << \omega_8 \ll \omega_9$ n'est pas vérifié
alors il faut calculer Gg précisement.
↳ Mathcad, Matlab, Tableur