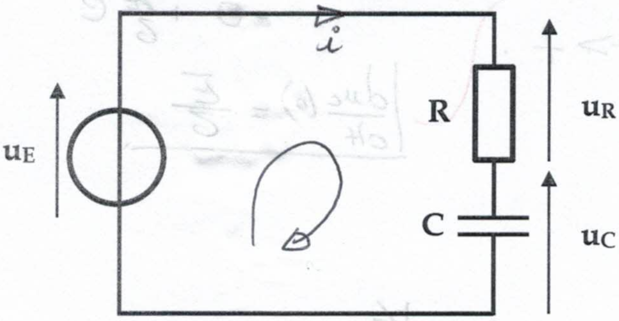
TD n°4 : Réponse temporelle d'un filtre du 1er ordre

I. Circuit RC série



① Loi des mailles.

$u_E(t) - u_R(t) - u_C(t) = 0$.

Equations instantanées sur les composants

$u_R(t) = R\cdot i(t)$.

$i(t) = C\,\frac{du_C}{dt}(t)$.

$\Leftrightarrow u_E(t) = R\cdot i(t) + u_C(t)$

On cherche à enlever $i(t)$ (équation avec u_E et u_C)

$\Leftrightarrow \boxed{u_E(t) = R\cdot C\,\frac{du_C}{dt}(t) + u_C(t).}$ $\Rightarrow \boxed{u_C(t) + RC\,\frac{du_C}{dt}(t) = E}$

② $u_C(t) = (u_C(0) - E)\,e^{-\frac{t}{RC}} + E$

$u_C(0) = 0 \Rightarrow \boxed{u_C(t) = E\left(1 - e^{-t/RC}\right)}$

③ $u_C(t) \xrightarrow[t \to +\infty]{} E(1-0) = E$

④ $u_C(t_1) = 63\% \cdot E = 0{,}63\cdot E$

$0{,}63\cdot E = E\left(1 - e^{-t_1/RC}\right)$

$e^{-\frac{t_1}{RC}} = 1 - 0{,}63 = 0{,}37$

$\ln\left(e^{-\frac{t_1}{RC}}\right) = \ln(0{,}37)$. $\Rightarrow -\frac{t_1}{RC} = \ln(0{,}37)$.

$t_1 = -RC\cdot\underbrace{\ln(0{,}37)}_{\simeq -1}$

$\boxed{t_1 = \tau = RC}$

⑤ $u_C(t_2) = 0{,}9\cdot E \Rightarrow 0{,}9 = \left(1 - e^{-t_2/\tau}\right) \Rightarrow e^{-\frac{t_2}{\tau}} = 0{,}1$.

$e^{\frac{t_2}{\tau}} = 10$.

$\boxed{t_2 = \tau \ln 10}$

$\boxed{t_2 = 2{,}3\cdot\tau}$

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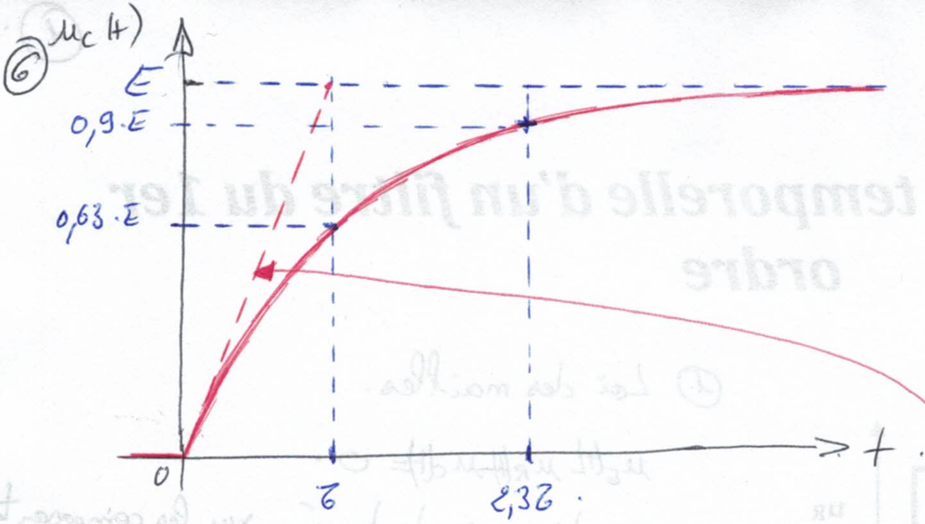


7 Equation de la tangente

$\Rightarrow \dfrac{du_C}{dt}(t)$

$u_C(t) = E\left(1 - e^{-t/\tau}\right)$.

$\dfrac{du_C}{dt}(t) = \dfrac{dE}{dt} - E\dfrac{d\left(e^{-t/\tau}\right)}{dt}$

$= 0 + \dfrac{E}{\tau} e^{-t/\tau}$.

$\boxed{\dfrac{du_C}{dt}(0) = \dfrac{E}{\tau}}$.

8 Cette fois $u_E(t) = 0$.

$\Rightarrow u_C(t) + RC\dfrac{du_C}{dt}(t) = 0$. $\quad K = 0$

Donc la solution est $u_C(t) = \left(u_C(0) - 0\right)e^{-t/\tau} + 0$.

$\boxed{u_C(t) = u_C(0)\, e^{-t/\tau}}$.

9 $u_C(0) = E$.

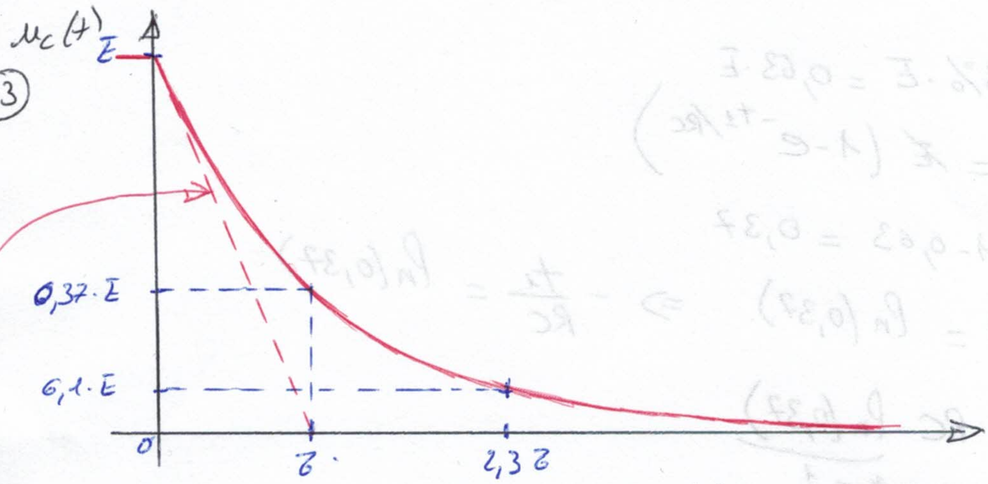
10 $\Rightarrow \boxed{u_C(t) = E\, e^{-t/\tau}}$. $\quad \boxed{u_C(t) \xrightarrow[t \to +\infty]{} 0}$.

11 Si condensateur déchargé de 63% $\Rightarrow u_C(t_3) = 0{,}37 \cdot E$.

$0{,}37 \cdot E = E\, e^{-t_3/\tau} \Rightarrow t_3 = \tau \cdot \ln\left(\dfrac{1}{0{,}37}\right) \simeq \tau$

12 Idem pour 90%. $\Rightarrow t_4 = \tau \ln\left(\dfrac{1}{0{,}1}\right) \simeq 2{,}3\tau$

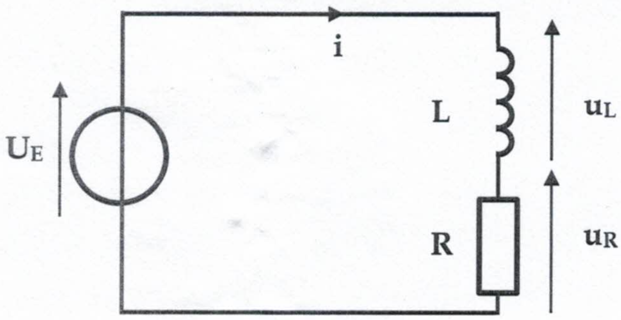
13



14 Tangente $\Rightarrow \dfrac{du_C}{dt}(t) = -\dfrac{E}{\tau} e^{-t/\tau}$.

$\dfrac{du_C}{dt}(0) = -\dfrac{E}{\tau}$ pente négative decroissance de E en un tps τ.

II. Circuit RL série



1. Loi des mailles.

$u_E(t) = u_L(t) + u_R(t)$.

loi d'Ohm inst.

$u_L(t) = L\frac{di}{dt}(t)$.

$u_R(t) = R\,i(t)$.

$\Leftrightarrow u_E(t) = L\frac{di}{dt}(t) + Ri(t)$.

$$\boxed{i(t) + \overbrace{\frac{L}{R}}^{\tau}\frac{di}{dt}(t) = \frac{u_E(t)}{R} = \frac{E}{R}}$$

2. $K = \frac{E}{R} \Rightarrow i(t) = \left(i(0) - \frac{E}{R}\right)e^{-\frac{t}{L/R}} + \frac{E}{R}$.

$i(0) = 0 \Rightarrow \boxed{i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right) \text{ avec } \tau = \frac{L}{R}}$.

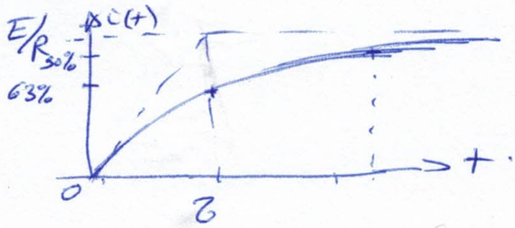
3. $i(t) \xrightarrow[t\to\infty]{} \frac{E}{R}$.

4. 5. idem II 4 et 5

63% à $t_1 = \tau$.

90% à $t_2 = 2,3\tau$.

6. Mˆ chose



7. $\frac{di}{dt}(0) = \frac{E/R}{\tau} = \frac{E/R}{L/R} = \frac{E}{L}$.

8. $u_E(t) = 0 \Rightarrow K = 0 \Rightarrow i(t) = (i(0) - 0)e^{-t/\tau}$ avec $\tau = L/R$.

9. $\boxed{i(t) = i(0)e^{-t/\tau} = \frac{E}{R}e^{-t/\tau}}$

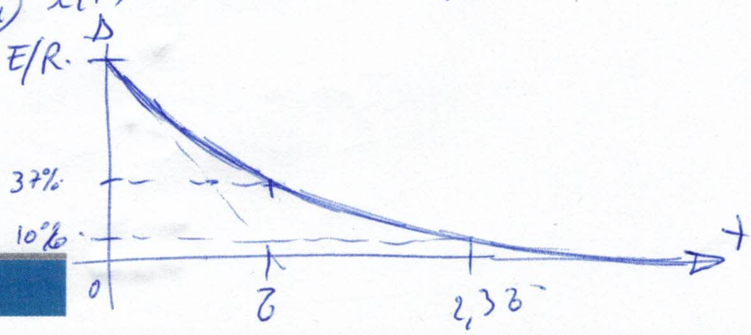
10. $i(t) \xrightarrow[t\to\infty]{} 0$.

11. 12. Idem II 11 et 12

Chute de 63% de i à $t = \tau$.

——— 90% de i à $t = 2,3\tau$.

13. 14. i(t)



$\frac{di}{dt}(t) = -\frac{E/R}{\tau}e^{-t/\tau}$.

$\frac{di}{dt}(0) = -\frac{E/R}{\tau} = -\frac{E/R}{L/R} = -\frac{E}{L}$